

A general strategy for the simultaneous classification of variables and objects in ecological data tables

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Abstract. A general iterative algorithm is proposed for the arrangement of data matrices by optimizing χ^2 , sum of squares or pooled entropy for blocks determined by clusters of variables and objects. The method includes *k*-means clustering and constrained block clustering as special cases. Possibilities for evaluating the resulting matrix include calculation of relative contributions to measures of block sharpness, comparison of partitions and construction of consensus tables. A new measure for comparing rearranged data matrices is suggested. The distributional properties of final results produced under different starting conditions are examined. The performance of the algorithm is tested on binary vegetation data from the Italian Alps.

Keywords: Block clustering; Chi-square; Consensus; Entropy; Iterative relocation; Partition; Sum of squares; Tabular sorting.

Introduction

Classifications of variables and objects are often used in the rearrangement of raw data tables to interpret row clusters in terms of column clusters and vice versa. Tabular sorting of matrices of species by sampling units (relevés) is a well-known example of this approach. Automated rearrangement methods include separate hierarchical clustering of species and sampling units through normal and 'inverse' association analysis (e.g. Lambert & Williams 1962) and incremental sum of squares agglomeration (e.g. Feoli & Orlóci 1979). Hartigan (1972) suggested a method which operates by divisions of row or column groups such that the decrease of sum of squares is maximized. Other, more sophisticated techniques require specification of various threshold values for removing outliers from the table in order to increase within-block homogeneity (e.g. Ceska & Roemer 1971; van der Maarel, Janssen &

Louppen 1978). The combined application of clustering and ordination procedures has also been suggested for improving the efficiency of data structuring procedures (e.g. Dale & Anderson 1973; Wildi 1989). All these methods provide solutions to the same general problem of creating classifications of objects and variables such that their interaction is best revealed.

In this paper we describe an iterative relocation method which is useful when the objective is simultaneous classification of variables and objects. The strategy uses three different criteria for measuring sharpness of block structure in the rearranged matrix, so that users can select the criterion most compatible with the actual data type and alternative data structuring methods. A dissimilarity measure for comparing rearranged data tables is also proposed.

Measures of block sharpness

For *p* clusters of *n* variables and *q* clusters of *m* objects, the rearranged matrix will have *pq* rectangular blocks. Measures of the distinctiveness of these blocks are required to find an optimum simultaneous classification of variables and objects. Selection of this measure is primarily determined by the data type used.

For binary data (e.g. presence-absence) the χ^2 statistic offers a possibility. It has been used for the a posteriori evaluation of block structure in rearranged phytosociological data tables by Feoli & Orlóci (1979). This statistic is given by the formula:

$$\chi_{(p,q)}^2 = \sum_{i=1}^p \sum_{j=1}^q \frac{(f_{ij} - \frac{f_{i.} f_{.j}}{f_{..}})^2}{\frac{f_{i.} f_{.j}}{f_{..}}} \quad (1)$$

where f_{ij} is the number of presences in the data block *ij* determined by variable group *i* and object group *j*. For a

structure-free table with equal numbers of presences for all blocks χ^2 will be zero, whereas a maximum value is reached in a situation where every group of objects is explained fully by the presence of one and only one group of variables (Fig. 1). For a random arrangement of values χ^2 is expected to be slightly higher than 0 (see Cramér 1946).

In case of *nominal* (multistate) characters, the information statistic expressing pooled within-block weighted entropy is straightforward:

$$H_{(p,q)} = \sum_{i=1}^p \sum_{j=1}^q \left(k_i k_j \log k_i k_j - \sum_{h=1}^s f_{hij} \log f_{hij} \right) \quad (2)$$

where k_i and k_j are the number of variables in group i and the number of objects in group j , respectively, s is the number of character states, and f_{hij} is the frequency of character state h in block ij . Measure (2) can also be applied to binary data, of course, but the resulting entropy will be independent of the coding of variables (i.e. it does not matter whether presences are coded by 1-s and absences by 0-s or vice versa). This is not so with the χ^2 statistic. In the binary case, therefore, choice between these two functions is crucial.

For variables measured on the *interval* and *ratio* scale (cf. Anderberg 1973, p. 27), the pooled within-block sum of squares criterion is used:

$$S_{(p,q)} = \sum_{i=1}^p \sum_{j=1}^q (x_{ij} - z_{ij})^2, \quad (3)$$

where z_{ij} denotes the mean of all values in the block containing x_{ij} . This is the criterion used by Hartigan (1972) in his divisive block clustering procedure.

Rearrangement of data

Optimal rearrangement of data matrices involves maximizing of (1), or minimizing of (2) or (3). The exact solution would be obtained by the examination of all possible classifications of n variables into p clusters over each of all classifications of m objects into q clusters. Given that the number of possible classifications of n items into two clusters is

$$\beta_{2(n)} = \frac{2^n - 2}{2!} \quad (4)$$

the number of possible classifications into $p(>2)$ non-empty groups is calculated using the following recursive formula:

a		b	
$\chi^2 = 0$		$\chi^2 = 54$	
1 - - -1 - - -	9	111 - - - - -	9
-1 -1 - - -11		111 - - - - -	
-1 - - -1 1 - -		111 - - - - -	
1 - - - -1 -1 -	9	- - - 111 - - -	9
- - - 1 - - 1 - -		- - - 111 - - -	
-11 -1 - - -1		- - - 111 - - -	
-1 - 11 - 1 - -	9	- - - - - 111	9
1 - - - -1 -1 -		- - - - - 111	
-1 - - - - -1		- - - - - 111	
9 9 9	27	9 9 9	27

Fig. 1. Examples for structure-free (a) and completely structured (b) blocked binary data matrices with similar row and column totals for the groups.

$$\beta_{p(n)} = \frac{p^n - p - \sum_{k=2}^{p-1} \binom{n}{k} k! \beta_{k(n)}}{p!} \quad (5)$$

The number of different arrangements of the matrix is thus $\beta_{p(n)} \beta_{q(m)}$. It means, for example, that for the reasonable values of $n = m = 100, p = q = 3$, the number of different blocked matrices to be examined would be $(3^{100} - 3 \cdot 2^{100} + 3)/36 = 7.4 \cdot 10^{93}$, making complete enumeration impossible. To approximate the true optimum, we therefore suggest a relatively fast iterative procedure based on the following algorithm.

1. Specify an initial partition for variables and objects into p and q clusters, respectively. This partition may be generated at random, derived from results of other cluster analyses, or defined arbitrarily based on preconceptions about the classifications.
2. Calculate measure of block sharpness χ_i^2, S_i or H_i for the starting configuration (i.e. $i = 0$).
3. Set $i = i + 1$. Find the variable or object whose relocation from its group to any other group maximizes

$$\Delta\chi^2 = \chi_i^2 - \chi_{i-1}^2, \quad (6)$$

or

$$\Delta S = S_{i-1} - S_i, \quad (7)$$

or

$$\Delta H = H_{i-1} - H_i. \quad (8)$$

This step requires examination of $n(p-1) + m(q-1)$ relocations.

4. If $\Delta\chi^2_{\max}, \Delta S_{\max}$ or ΔH_{\max} is non-positive, go to Step 6, since further improvement of block structure is no

longer possible by relocating single data vectors. Otherwise go to next step.

5. Examine if the relocation of the object or variable associated with the maximum would result in an empty cluster. If yes, go to Step 6. Otherwise perform the relocation and go to Step 3.

6. Produce the resulting rearranged matrix.

Note that initializing with random blocks is likely to require more steps to reach the stable stage than when the initial arrangement is created by some objective data structuring procedure. This is especially true if, for example, minimization of S is started from classifications obtained by incremental sum of squares agglomeration, i.e. the methods apply similar clustering criteria.

As with other iterative procedures, the final result which cannot be further improved by a single relocation usually depends on the initial classifications; i.e. the iterations may converge to any one of several different stable solutions ('local optima'). Therefore, a single analysis may end up with a reasonably optimal rearrangement, but there is a slight chance that the final block structure obtained from randomly generated clusters is not satisfactory. To be certain that the worst cases are eliminated, the iterations should be repeated many times using different starting configurations. If the analysis stops to avoid empty clusters, then we may assume that data structuring with a lower number of clusters will be more efficient.

The sequence of clusters in the rearranged matrix is undefined by the above algorithm. If interest lies in obtaining a diagonal arrangement for situations when a strong underlying gradient is suspected, analysis of concentration (Feoli & Orlóci 1979) may be used to determine positions of groups in the sequence.

Relationship to k-means clustering

The above algorithm contains k -means clustering (Hartigan 1975, p. 85) as a special case when p or q is set to 1 for the sum of squares criterion. Partitioning only variables or objects based on the minimum pooled within-group entropy is an analogous procedure. Of course, the use of the χ^2 statistic requires that $p, q > 1$, otherwise $\chi^2 = 0$.

Constrained block clustering

Another special case occurs when the above algorithm is modified so that the initial classification of columns (or rows) remains unchanged, and only rows (or columns) are allowed to move from one cluster to another. Such a method is useful if we wish to find a classification of variables which provides most optimal

blocks for a given classification of objects, or vice versa. For example, constrained block clustering finds species groups that best explain a classification of relevés obtained as a consensus of many alternative classifications. Of course, the constraint that one of the classifications remains intact during iterations may result in a rearranged block structure which is far from optimal, particularly when the constraining classification is not itself optimal.

Evaluation of results

Relative contributions

The analysis of variable/object interactions rarely ends with the optimization of block structure, since one is often interested in evaluating the relative contribution of individual objects and variables to the configuration. This can be used to select rows and columns which conflict with the block structure. These may then be declared as outliers and removed from the data. Consequently, such an evaluation can help identify variables and objects that are in close agreement with the classifications.

Contributions of variables and objects are determined by calculating the distinctiveness of blocks such that one variable or object is removed from the data matrix at a time. While the new value may be larger or smaller than the original for the χ^2 statistic, it is always smaller for H and S .

Another possible evaluation method is to examine the contribution of blocks to the total χ^2 , entropy or sum of squares. Exceedingly high percentages associated with a particular row or column group suggest that its division may produce a more efficient data structuring.

Comparisons

The investigator may be interested in comparisons of species and relevé classifications obtained by different criteria of block structuring. The comparison of classifications in the final table to classifications obtained by phytosociological methods is another possibility. Dissimilarity indices for comparing partitions may be used for this purpose. In this paper we use the minimum number of relocations of objects necessary to convert one partition to the other (denoted by M) as a coefficient of partition agreement ('minimum number of divisions, mergences and transfers', Day 1981). Division of this number by the possible maximum, $\max(M)$, gives a dissimilarity measure denoted by μ ('misclassification index', Podani 1986).

Comparison of two alternative, rearranged data

matrices based on species and relevé classifications separately provides two coefficients that are not as easy to interpret as a dissimilarity of the tables themselves. Therefore, we propose a single measure for the direct comparison of rearranged data tables. We assume that we have two rearranged matrices, X_i and X_j , both with p clusters for n species (rows, variables) and q clusters for m relevés (columns, objects). The basic idea is to count the number of data values to be relocated from one block to another to reproduce X_i from X_j or vice versa. This number is determined by the comparison of the two species partitions and the two relevé partitions using the M metric, which yields $M_{ij(\text{rows})}$ and $M_{ij(\text{cols})}$, respectively. From these measures we obtain the number of data items to be relocated by the formula:

$$K = mM_{ij(\text{rows})} + nM_{ij(\text{cols})} - M_{ij(\text{rows})}M_{ij(\text{cols})} \quad (9)$$

Division of K with the possible maximum yields the dissimilarity measure sought:

$$\kappa = \frac{mM_{ij(\text{rows})} + nM_{ij(\text{cols})} - M_{ij(\text{rows})}M_{ij(\text{cols})}}{m \max M_{ij(\text{rows})} + n \max M_{ij(\text{cols})} - \max(M_{ij(\text{rows})}) \max(M_{ij(\text{cols})})} \quad (10)$$

This measure ranges from 0 to 1, indicating complete identity and maximum dissimilarity, respectively. The maxima necessary for calculating μ and κ are determined by maximizing the sum of non-diagonal values in cross-classification tables (Podani 1986).

Consensus blocks

Given $r \geq 2$ alternative rearranged tables for the same variables and objects (obtained, for example, by minimizing functions 1-3 for binary scores), one may be interested in assessing the agreements among the alternative results. The generation of consensus partitions separately for rows and for columns offers a possibility to create such a 'synthetic' table. Two consensus methods appear meaningful in this context. The strict consensus classes contain objects that are consistently classified together in all cases, so that the assignment of constituting objects is unambiguous. Another possibility is the application of the majority rule consensus approach (Day 1988; Podani 1989a) in which two objects are considered to be in the same class if they are classified together in most cases, even though they may be separated in some cases. The global optimization method suggested by Podani (1989b) may be used to generate both strict and majority consensus partitions. The advantage of consensus generation is that the 'average' result may be more acceptable than any of the starting alternatives, since the existence of consensus clusters is justified by all or the majority of methods of tabular rearrangement.

Illustrative examples

Sample data

The data set for illustrating the method is taken from a syntaxonomic study in alpine vegetation in Italy (Giacomini & Pignatti 1955). The data include cover-abundance scores for 69 species in 50 relevés. The data have been used by Feoli & Zuccarello (1988) to show the usefulness of fuzzy set theory in revealing syndynamic information (see also Banyikwa, Feoli & Zuccarello 1990). Using the Braun-Blanquet methodology, four vegetation types were recognized: 1. *Polytrichetum* (relevés 1-11); 2. *Salicetum herbaceae salicetosum* (rel. 12-24). 3. *Salicetum herbaceae alchemilletosum* (rel. 25-44); and 4. *Hygrocurvuletum* (rel. 45-50). These communities represent stages of a successional sere which is controlled by snow depth and persistence (Feoli & Zuccarello 1988). We used five species groups in order to identify species groups characterizing each type and another group without preference of any types.

Methods

The sample data set was subjected to an extensive computerized study in order to find optimum block structure for four relevé groups and five species groups based only on presence-absence data and all the three measures of block sharpness. Computer program REBLOCK has been written in Fortran for block clustering (see Appendix). For each measure of block sharpness, three different types of analysis were performed: a. In the first, the optimization started from randomly generated groups for both the species and the relevés (termed RR analysis in the sequel). b. The second type of analysis began with randomly generated groups for species, but the initial partition of relevés was the phytosociological classification specified in the previous section (called RT analysis). c. The third one was constrained block clustering, i.e. the starting classification of species was random but the phytosociological classification of relevés remained unchanged during the iterations (RTC analysis).

Each analysis was repeated 100 times in order to find an approximation to the frequency distribution of the resulting measures of block sharpness. The total number of analyses was thus 900. This facilitates not only the evaluation of a particular data structuring method, but also the comparison of the performance of different techniques. The sequence of groups within the rearranged tables was defined by the first axis obtained by Analysis of Concentration (Feoli & Orlóci 1979). The program was run on the CONVEX mainframe computer

of the International Centre of Theoretical Physics in Trieste.

Based on the most optimal results obtained for each of the three criteria, strict and majority consensus classes were constructed (program MINGFC, Podani 1989b) in order to illustrate agreements among the results and to find species and relevé groups with the most consistent classification properties. Partitions were compared using the measures offered by program PARCOM in the SYN-TAX IV package (Podani in press).

Results and Discussion

Frequency distributions

The frequency histograms of optimal criterion values obtained in repeated runs of program REBLOCK demonstrate that the distributions are more or less skewed (Figs. 2-4), revealing an obvious tendency towards the optimum. The broadest range of outcomes was produced by the RR analysis for all cases (diagram A in each of Figs. 2-4). This is an obvious consequence of using randomized initial clusters. The second widest range resulted from the RT analysis. Initialization by a fixed classification of relevés considerably decreased the probability of 'worst case' outcomes. Constrained block clustering (RTC) yielded an even narrower range which is most striking for the χ^2 statistic (Fig. 3c).

In every distribution, the optimal results were reached only in a few runs of the program whereas the most likely values were suboptimal. In this regard, the χ^2 statistic outperforms the other two. The highest χ^2 was reached 11 times in the RR analysis, and 5 times in the RT analysis. Optimization of S was most successful in the RR analysis whereas minimization of entropy was almost equally successful in the RR and RT analyses. It is concluded that analyses beginning with a suboptimal input classification of relevés (such as the subjective phytosociological classification used in this example) may lead to a result which is as good as, if not better than results obtained from iterations starting from fully randomized initial clusters.

The histograms clearly show that the rearrangement of data is least acceptable if the classification of relevés is fixed, even if it is quite close to the optimal partition.

The question arises whether the most optimal outcome is produced by the RR or the RT analysis. There is no general answer to this, because the efficiency of RT analyses will always depend on the initial classification. Therefore, one is advised to try both strategies, especially if a reasonable starting hypothesis is available concerning the classification of relevés. Constrained clustering, in view of the above results, is applicable

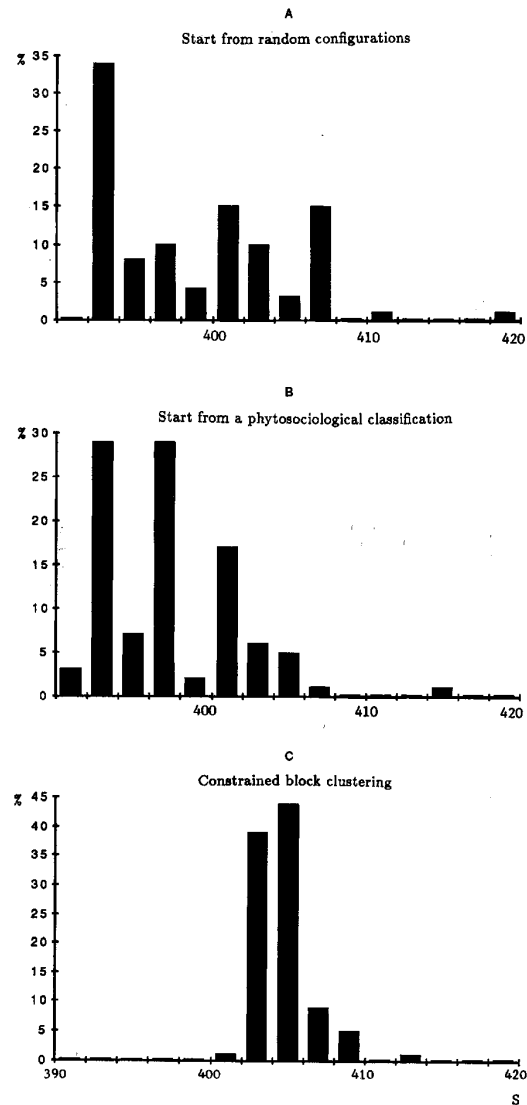


Fig. 2. Frequency distributions of results for sum of squares block clustering of alpine vegetation data. Each diagram is based on 100 analyses.

only if one has a good reason to use a fixed classification.

Optimal structured tables

Upon the first inspection of optimized binary data matrices (Tables 1-3) one easily observes that the use of different optimality criteria does not lead to the same classification. This is expected for most clustering problems, unless there is an obvious group structure in the

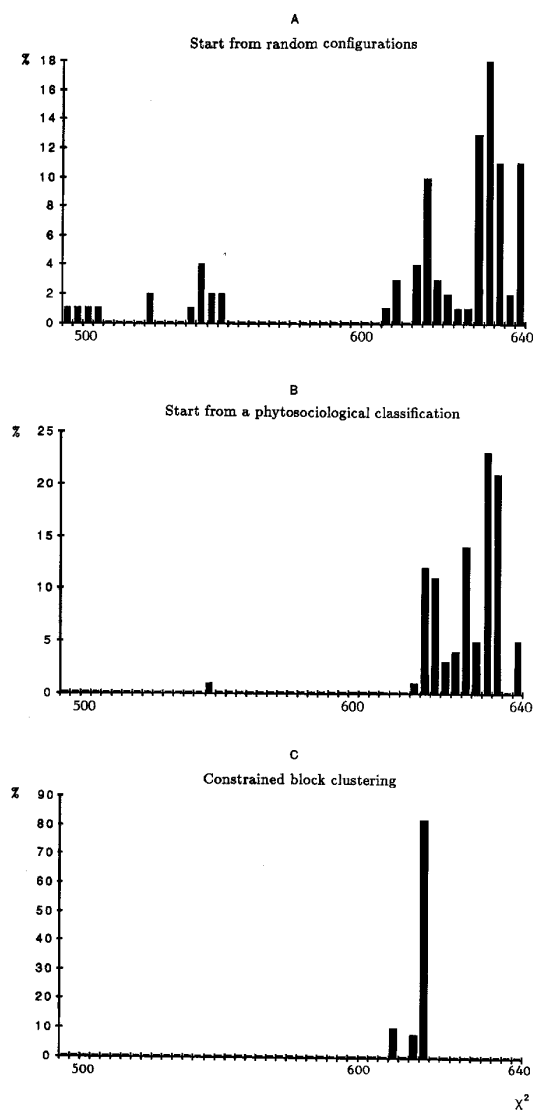


Fig. 3. Frequency distributions of results for χ^2 -based block clustering of alpine vegetation data. Each diagram is based on 100 analyses.

data (such as in Fig. 1). The sum of squares criterion produced blocks that do not occur in the other two cases. On the other hand, optimization of χ^2 and entropy produced an identical cluster for species and another for relevés (last row and column of blocks in Tables 2-3). This species group almost perfectly characterizes community type *Hygrocurvuletum*, with only *Pogonatum alpinum* occurring in a relevé from another type. This species group was not recognized when the sum of squares criterion was applied (Table 1), illustrating a

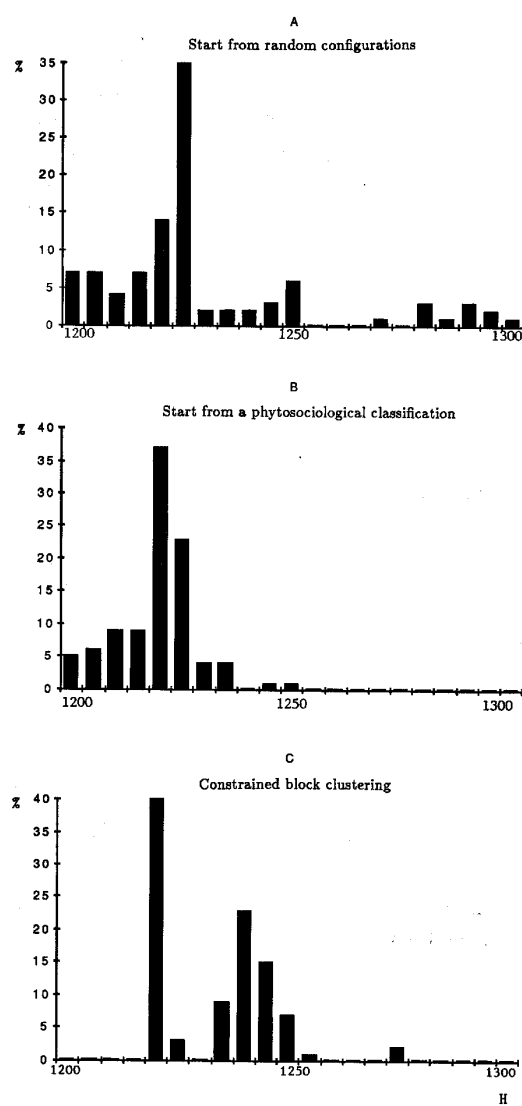


Fig. 4. Frequency distributions of results for entropy based block clustering of alpine vegetation data. Each diagram is based on 100 analyses.

tendency of this method to classify species according to commonness and rarity rather than association. This finding confirms the general view that species groups obtained by incremental sum of squares clustering are usually mere reflections of species frequency.

For partitions of species, the above conclusions are confirmed by the objective comparisons. The χ^2 statistic and entropy produced considerably similar species clusters ($M = 11$, $\mu = 0.2$), whereas the sum of squares criterion led to a classification more dissimilar from the

other two ($M = 23$, $\mu = 0.41$ and $M = 25$, $\mu = 0.45$, respectively). For relevés, the sum of squares and entropy criteria produced very close classifications ($M = 5$, $\mu = 0.13$), and both are far from the phytosociological classification ($M = 18$, $\mu = 0.48$ and $M = 13$, $\mu = 0.35$, respectively) and the χ^2 -based partition ($M = 18$, $\mu = 0.48$ and $M = 17$, $\mu = 0.46$, respectively). On the other hand, the phytosociological classification exhibits the highest similarity to the partition obtained by minimizing χ^2 ($M = 8$, $\mu = 0.2$).

The classifications of relevés do not correspond perfectly to the phytosociological types recognized by Giacomini & Pignatti (1955). It is obvious from the results that the two subassociations of the *Salicetum* are very similar and strictly related from a syndynamic viewpoint. In fact, they are not separable at the presence-absence level. A slight overlap between *Polytrichetum* and *Salicetum* is also clear. The *Hygrocurvuletum* appears to be most clearly defined, although the sum of squares analysis did not separate it completely from the *Salicetum*.

Although the number of species clusters was set to five in order to find characteristic species groups, most clusters do not exhibit exclusive preferences (except for the last species group in Tables 2-3, which almost perfectly characterizes the *Hygrocurvuletum* community). Indeed, the first four groups confirm the existence of the continuity of types along the gradient in all tables. For example, the first group has a peaked dominance in the first relevé group, whereas the second characterizes both the first and the second relevé groups.

The species groups are not identical and, what is more striking from an ecological viewpoint, some species are clustered together even though they never co-occur in the relevés (for example, *Luzula lutea* and *Cardamine resedifolia*). This is because in this kind of clustering groups of relevés are described in terms of species groups and vice versa. In this regard, the two species mentioned above exhibit a similar behavior. The overall comparison of tables using the κ measure indicates that the sum of squares criterion and entropy produced the most similar tables ($\kappa = 0.42$), although none of the blocks are identical. The entropy and the χ^2 criteria did produce similar blocks, but the dissimilarity of the matrices is a little higher ($\kappa = 0.47$). Since sum of squares and χ^2 produced very different clusters for both species and relevés, the high dissimilarity of the matrices is not surprising ($\kappa = 0.625$).

Consensus blocks

The optimal rearranged matrices (Tables 1-3) were further analyzed to reveal underlying properties of the data jointly recognized by all the three measures of

block sharpness. The majority rule consensus matrix (not shown) with five species groups and four relevé groups, however, does not give substantial new information on the species/site interactions. On the contrary, the rearranged matrix based on the strict consensus clusters, excluding groups with 1 - 3 members only, reveals a very interesting property of the data (Table 4). It indicates a 'double' diagonal structure, one diagonal associated with the frequent species, and the other for the rare species. The two types found at the endpoint of the gradient are very well separated in the table, whereas the two subassociations of the *Salicetum* comprise three groups.

Conclusions

The block clustering strategy proposed here requires a definition of the number of clusters for variables and relevés, so it is recommended for application after some information is already available on group structure. The relocation algorithm treats rows and columns as items with equal importance, therefore the method works only if the notion of dissimilarity is similar for both species and relevés. The iterations do not arrange groups into a meaningful order but, if necessary, such an ordering may be obtained by analysis of concentration or, in simple cases, by manual rearrangement. The results demonstrated that this combined approach reveals inherent properties of the data that are not apparent otherwise (for example, the double diagonal structure of the strict consensus table).

Statistical ecologists now agree that there is no clustering method that gives superior results under all circumstances. Of course, this statement is even more valid for the technique presented here. This is further complicated by the fact that iterative procedures do not necessarily yield the true optimum. The analyses involve a heavy computational load, because the final results are influenced by the starting partitions. Nevertheless, the wide availability of fast computers facilitates such investigations. However, no definite answer can be given to the question regarding the number of iterations required to reach a fairly good result, because it is case-dependent. Like in many situations in ecology, examination of distributions of outcomes gives more information on the properties of data than a single result. Considerable spread of values in the histograms suggests that the species and relevés are less classifiable than when the final block structure is largely independent of the starting configuration. Obviously, a more extensive study using data with different properties is required to examine this problem more deeply.

The very small differences among the optimal and

Table 1. Rearranged table for the alpine vegetation data with sum of squares of blocks minimized in 300 analyses of presence-absence scores, four relevé groups and five species groups. $S_{(5,4)} = 390.64$.

	Relevés			
	0000000011111 12356789012378	011112222222233333 4456012456789012456	333344444 378901234	12444445 93567890
<i>Anthelia juratzkana</i>	1111111111	1111111111	1111111111	1111111111
<i>Webera commutata</i>	1111111111	1111111111	1111111111	1111111111
<i>Gentiana bavarica</i>	1111111111	1111111111	1111111111	1111111111
<i>Sedum alpestre</i>	1111111111	1111111111	1111111111	1111111111
<i>Cerastium cerastoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Taraxacum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa supina</i>	1111111111	1111111111	1111111111	1111111111
<i>Ranunculus glacialis</i>	1111111111	1111111111	1111111111	1111111111
<i>Cardamine alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum sexangulare</i>	1111111111	1111111111	1111111111	1111111111
<i>Arenaria biflora</i>	1111111111	1111111111	1111111111	1111111111
<i>Gnaphalium supinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Salix herbacea</i>	1111111111	1111111111	1111111111	1111111111
<i>Veronica alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Chrysanthemum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa minor</i>	1111111111	1111111111	1111111111	1111111111
<i>Soldanella pusilla</i>	1111111111	1111111111	1111111111	1111111111
<i>Alchemilla pentaphyllea</i>	1111111111	1111111111	1111111111	1111111111
<i>Sagina saginoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Carex curvula</i>	1111111111	1111111111	1111111111	1111111111
<i>Agrostis rupestris</i>	1111111111	1111111111	1111111111	1111111111
<i>Ochrolechia sp.</i>	1111111111	1111111111	1111111111	1111111111
<i>Cladonia pyxidata</i>	1111111111	1111111111	1111111111	1111111111
<i>Euphrasia minima</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum juniperinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Sibbaldia procumbens</i>	1111111111	1111111111	1111111111	1111111111
<i>Cladonia rangiferina</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum piliferum</i>	1111111111	1111111111	1111111111	1111111111
<i>Leontodon pyrenaicus</i>	1111111111	1111111111	1111111111	1111111111
<i>Polygonum viviparum</i>	1111111111	1111111111	1111111111	1111111111
<i>Ligusticum mutellina</i>	1111111111	1111111111	1111111111	1111111111
<i>Potentilla aurea</i>	1111111111	1111111111	1111111111	1111111111
<i>Sieversia montana</i>	1111111111	1111111111	1111111111	1111111111
<i>Nardus stricta</i>	1111111111	1111111111	1111111111	1111111111
<i>Euphrasia drosocalyx</i>	1111111111	1111111111	1111111111	1111111111
<i>Luzula spicata</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria islandica</i>	1111111111	1111111111	1111111111	1111111111
<i>Webera carinata</i>	1111111111	1111111111	1111111111	1111111111
<i>Kiaeria starkei</i>	1111111111	1111111111	1111111111	1111111111
<i>Pleuroclada albescens</i>	1111111111	1111111111	1111111111	1111111111
<i>Gymnomitrium varians</i>	1111111111	1111111111	1111111111	1111111111
<i>Dicranum falcatum</i>	1111111111	1111111111	1111111111	1111111111
<i>Carex lachenalii</i>	1111111111	1111111111	1111111111	1111111111
<i>Stereocaulon coralloides</i>	1111111111	1111111111	1111111111	1111111111
<i>Dicranoweisia crispula</i>	1111111111	1111111111	1111111111	1111111111
<i>Pogonatum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Phyteuma hemisphaericum</i>	1111111111	1111111111	1111111111	1111111111
<i>Minuartia sedoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Luzula lutea</i>	1111111111	1111111111	1111111111	1111111111
<i>Cardamine resedifolia</i>	1111111111	1111111111	1111111111	1111111111
<i>Rhacomitrium ericoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Senecio carniolicus</i>	1111111111	1111111111	1111111111	1111111111
<i>Hieracium glanduliferum</i>	1111111111	1111111111	1111111111	1111111111
<i>Thamnochloa vermicularis</i>	1111111111	1111111111	1111111111	1111111111
<i>Alectoria ochroleuca</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria cucullata</i>	1111111111	1111111111	1111111111	1111111111
<i>Veronica bellidioides</i>	1111111111	1111111111	1111111111	1111111111
<i>Festuca halleri</i>	1111111111	1111111111	1111111111	1111111111
<i>Silene acaulis ssp. exscapa</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Oreocloa disticha</i>	1111111111	1111111111	1111111111	1111111111
<i>Primula oenosis</i>	1111111111	1111111111	1111111111	1111111111
<i>Avenula versicolor</i>	1111111111	1111111111	1111111111	1111111111
<i>Soldanella alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Erigeron uniflorus</i>	1111111111	1111111111	1111111111	1111111111
<i>Phyteuma globulariifolium</i>	1111111111	1111111111	1111111111	1111111111
<i>Coelocaulon aculeatum</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria nivalis</i>	1111111111	1111111111	1111111111	1111111111
<i>Paraleucobryum nerve</i>	1111111111	1111111111	1111111111	1111111111

Table 2. Rearranged table for the alpine vegetation data with χ^2 minimized in 300 analyses of presence-absence scores, four relevé groups and five species groups. $\chi^2_{(5,4)} = 634.8$.

	Relevés			
	11 235678901	1111122223 1423467801470	112222233333333344444 5923568912345678901234	444445 567890
<i>Anthelia juratzkana</i>	1111111111	1111111111	1111111111	1111111111
<i>Webera commutata</i>	1111111111	1111111111	1111111111	1111111111
<i>Webera carinata</i>	1111111111	1111111111	1111111111	1111111111
<i>Pleuroclada albescens</i>	1111111111	1111111111	1111111111	1111111111
<i>Gymnomitrium varians</i>	1111111111	1111111111	1111111111	1111111111
<i>Dicranum falcatum</i>	1111111111	1111111111	1111111111	1111111111
<i>Gentiana bavarica</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa supina</i>	1111111111	1111111111	1111111111	1111111111
<i>Ranunculus glacialis</i>	1111111111	1111111111	1111111111	1111111111
<i>Carex lachenalii</i>	1111111111	1111111111	1111111111	1111111111
<i>Stereocaulon coralloides</i>	1111111111	1111111111	1111111111	1111111111
<i>Dicranoweisia crispula</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum sexangulare</i>	1111111111	1111111111	1111111111	1111111111
<i>Arenaria biflora</i>	1111111111	1111111111	1111111111	1111111111
<i>Veronica alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Sedum alpestre</i>	1111111111	1111111111	1111111111	1111111111
<i>Cerastium cerastoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Taraxacum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Cardamine alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Kiaeria starkei</i>	1111111111	1111111111	1111111111	1111111111
<i>Gnaphalium supinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Salix herbacea</i>	1111111111	1111111111	1111111111	1111111111
<i>Soldanella pusilla</i>	1111111111	1111111111	1111111111	1111111111
<i>Sibbaldia procumbens</i>	1111111111	1111111111	1111111111	1111111111
<i>Alchemilla pentaphyllea</i>	1111111111	1111111111	1111111111	1111111111
<i>Sagina saginoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Chrysanthemum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa minor</i>	1111111111	1111111111	1111111111	1111111111
<i>Agrostis rupestris</i>	1111111111	1111111111	1111111111	1111111111
<i>Ochrolechia sp.</i>	1111111111	1111111111	1111111111	1111111111
<i>Cladonia pyxidata</i>	1111111111	1111111111	1111111111	1111111111
<i>Euphrasia minima</i>	1111111111	1111111111	1111111111	1111111111
<i>Luzula lutea</i>	1111111111	1111111111	1111111111	1111111111
<i>Cardamine resedifolia</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum juniperinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Rhacomitrium ericoides</i>	1111111111	1111111111	1111111111	1111111111
<i>Carex curvula</i>	1111111111	1111111111	1111111111	1111111111
<i>Cladonia rangiferina</i>	1111111111	1111111111	1111111111	1111111111
<i>Polytrichum piliferum</i>	1111111111	1111111111	1111111111	1111111111
<i>Leontodon pyrenaicus</i>	1111111111	1111111111	1111111111	1111111111
<i>Polygonum viviparum</i>	1111111111	1111111111	1111111111	1111111111
<i>Phyteuma hemisphaericum</i>	1111111111	1111111111	1111111111	1111111111
<i>Ligusticum mutellina</i>	1111111111	1111111111	1111111111	1111111111
<i>Potentilla aurea</i>	1111111111	1111111111	1111111111	1111111111
<i>Sieversia montana</i>	1111111111	1111111111	1111111111	1111111111
<i>Nardus stricta</i>	1111111111	1111111111	1111111111	1111111111
<i>Euphrasia drosocalyx</i>	1111111111	1111111111	1111111111	1111111111
<i>Luzula spicata</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria islandica</i>	1111111111	1111111111	1111111111	1111111111
<i>Pogonatum alpinum</i>	1111111111	1111111111	1111111111	1111111111
<i>Senecio carniolicus</i>	1111111111	1111111111	1111111111	1111111111
<i>Hieracium glanduliferum</i>	1111111111	1111111111	1111111111	1111111111
<i>Thamnochloa vermicularis</i>	1111111111	1111111111	1111111111	1111111111
<i>Alectoria ochroleuca</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria cucullata</i>	1111111111	1111111111	1111111111	1111111111
<i>Veronica bellidioides</i>	1111111111	1111111111	1111111111	1111111111
<i>Festuca halleri</i>	1111111111	1111111111	1111111111	1111111111
<i>Silene acaulis ssp. exscapa</i>	1111111111	1111111111	1111111111	1111111111
<i>Poa alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Oreocloa disticha</i>	1111111111	1111111111	1111111111	1111111111
<i>Primula oenosis</i>	1111111111	1111111111	1111111111	1111111111
<i>Avenula versicolor</i>	1111111111	1111111111	1111111111	1111111111
<i>Soldanella alpina</i>	1111111111	1111111111	1111111111	1111111111
<i>Erigeron uniflorus</i>	1111111111	1111111111	1111111111	1111111111
<i>Phyteuma globulariifolium</i>	1111111111	1111111111	1111111111	1111111111
<i>Coelocaulon aculeatum</i>	1111111111	1111111111	1111111111	1111111111
<i>Cetraria nivalis</i>	1111111111	1111111111	1111111111	1111111111
<i>Paraleucobryum nerve</i>	1111111111	1111111111	1111111111	1111111111

Table 3. Rearranged table for the alpine vegetation data with entropy minimized in 300 analyses of presence-absence scores, four relevé groups and five species groups. $H_{(5,4)} = 1209.38$.

	Relevés				
	1111 1234567890127	1111122222222333333 34567890123456789012456	333344444 378901234	444445	567890
<i>Webera comutata</i>	1111-1111				
<i>Webera carinata</i>	1-1-1-1				
<i>Kiaeria starkei</i>					
<i>Pleuroclada albescens</i>	1-1-1-1				
<i>Gymnomitrium varians</i>	1-1-1-1				
<i>Dicranum falcatum</i>	1-1-1-1				
<i>Gentiana bavarica</i>	1-1-1-1				
<i>Poa supina</i>	1-1-1-1				
<i>Ranunculus glacialis</i>	1-1-1-1				
<i>Carex lachenalii</i>	1-1-1-1				
<i>Stereocaulon coralloides</i>	1-1-1-1				
<i>Dicranoweisia crispula</i>	1-1-1-1				
<i>Lucula lutea</i>	1-1-1-1				
<i>Cardamine resedifolia</i>	1-1-1-1				
<i>Rhaconitrium ericoides</i>	1-1-1-1				
<i>Polytrichum sexangulare</i>	1111111111	11111-1-1-11-1-1-1	1-1-11		
<i>Anthelia juratzkana</i>	11-111111				
<i>Arenaria biflora</i>	1111111111	11-11-1-1-1-11111-1	1		
<i>Veronica alpina</i>	1-111111111	11-11-1-1-1-1-1-1-1-1	11-1-1		
<i>Sedum alpestre</i>	1-1-1111	111111-1-1-1-1-1-1-1-1-1			
<i>Sagina saginoides</i>	1-1-1-1	11-11-1-1-11-1111-1			
<i>Cerastium cerastioides</i>	11-1-1-11	11-11-1-1-1-1-1-1-1			
<i>Taraxacum alpinum</i>	1-1-1-1-11	11-11-1-1-1-1-1-1-1-1			
<i>Cardamine alpina</i>	1-1-1-1-11	11-1-1-1-1-1-1-1-1			
<i>Gnaphalium supinum</i>	1111-111111	1111111-1111111111	111111111		
<i>Salix herbacea</i>	11-1-1111	1111111111-1111-1-1-1-1	111111		
<i>Soldanella pusilla</i>	1-1-1-1	1-1-1-1-1111111111	111111111		
<i>Alchemilla pentaphyllea</i>	1-1-1-1	1-1-1-1-1111111111	111111111		
<i>Chrysanthemum alpinum</i>	1111111111	111111111-111111111	11-111-1		
<i>Poa minor</i>	11-1-1111	1111111	1111111111		
<i>Carex curvula</i>	1-1-1-1	11-1-1-1111-1111-1	11111111		
<i>Agrostis rupestris</i>	1-1-1-1	11-11-11-11-1-1-1-1-1-1	11-1-1-1		
<i>Ochrolechia sp.</i>	111111	111-11-1-1-1-1-1-1-111111			
<i>Polytrichum juniperinum</i>	1-11-1-1-1-1-1-1-1111	1-11111			
<i>Sibbaldia procumbens</i>	1-1-1-1	11-11111-1-1-1-1-1-1	1111-111		
<i>Cladonia pyxidata</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11111-1-1		
<i>Cladonia rangiferina</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11-111		
<i>Polytrichum piliferum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-11111-1		
<i>Leontodon pyrenaicus</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	111111		
<i>Polygonum viviparum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11-11-1		
<i>Phyteuma hemisphaericum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Ligusticum mutellina</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Potentilla aurea</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11111111		
<i>Stevensia montana</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-111111		
<i>Nardus stricta</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11111-1		
<i>Minuartia sedoides</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Euphrasia minima</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Euphrasia drosocalyx</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Lucula spicata</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Cetraria islandica</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	111111		
<i>Pogonatum alpinum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Senecio carniolicus</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1111		
<i>Hieracium glanduliferum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Thamnolia vermicularis</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Alectoria ochroleuca</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Cetraria cucullata</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Veronica bellidioides</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Festuca halleri</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Silene acaulis ssp. exscapa</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Poa alpina</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1111		
<i>Oreocloa disticha</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	111-11		
<i>Primula oenosis</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Avenula versicolor</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Soldanella alpina</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Erigeron uniflorus</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Phyteuma globularifolium</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		
<i>Coleocaulon aculeatum</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	11-1-1		
<i>Cetraria nivalis</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	111-1		
<i>Paraleucobryum enerve</i>	1-1-1-1	1-1-1-1-1-1-1-1-1-1-1-1-1-1	1-1-1-1		

Table 4. Strict consensus block structure for the alpine vegetation data obtained from rearranged matrices in Tables 1-3.

	Relevés				
	11 235678901	1122223 4601470	12222233333 52568912456	333344444 378901234	444445 567890
<i>Gnaphalium supinum</i>	11-1111	111111	1-11111111	11111111	1-1-1
<i>Salix herbacea</i>	1-1-1-1	1111-1	11-11-1-1	1-1-1-1	111111
<i>Chrysanthemum alpinum</i>	11111111	1111-11	11-11111111	11-111-1	1-1111
<i>Poa minor</i>	11-1-111	111111	1111111111	11111111	
<i>Soldanella pusilla</i>	1-1-1-1	1-1-1-1	1-11111111	11111111	1-1-1
<i>Alchemilla pentaphyllea</i>	1-1-1-1	11-1-1	1-11111111	11111111	1-1-1
<i>Agrostis rupestris</i>	1-1-1-1	1111-1	11-1-1-1-1	1-1-1-1	1-1-1
<i>Ochrolechia sp.</i>	1-1-1-1	1111-1	1-1-1-1-1	1-11111	
<i>Polytrichum juniperinum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-11111	
<i>Cladonia rangiferina</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-111-1	1-1-1
<i>Polytrichum piliferum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1111-1	1-1-1
<i>Leontodon pyrenaicus</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-11111	1-1-1
<i>Polygonum viviparum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1-1	1-1111
<i>Ligusticum mutellina</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-111111	1-1-1
<i>Potentilla aurea</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-111111	1-1-1
<i>Stevensia montana</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-111111	1-1-1
<i>Nardus stricta</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1111-1	1-1-1
<i>Euphrasia drosocalyx</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Lucula spicata</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cetraria islandica</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1111-1	111111
<i>Webera comutata</i>	11-1111	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Gentiana bavarica</i>	1-1111	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Poa supina</i>	1-111-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Ranunculus glacialis</i>	1-111-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Webera carinata</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Pleuroclada albescens</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Gymnomitrium varians</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Dicranum falcatum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Carex lachenalii</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Stereocaulon coralloides</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Dicranoweisia crispula</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Sedum alpestre</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cerastium cerastioides</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Taraxacum alpinum</i>	1-1111	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cardamine alpina</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Kiaeria starkei</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Lucula lutea</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cardamine resedifolia</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Rhaconitrium ericoides</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Pogonatum alpinum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Senecio carniolicus</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Hieracium glanduliferum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Thamnolia vermicularis</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Alectoria ochroleuca</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cetraria cucullata</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Veronica bellidioides</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Festuca halleri</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Silene acaulis ssp. exscapa</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Poa alpina</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Oreocloa disticha</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Primula oenosis</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Avenula versicolor</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Soldanella alpina</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Erigeron uniflorus</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Phyteuma globularifolium</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Coleocaulon aculeatum</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Cetraria nivalis</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1
<i>Paraleucobryum enerve</i>	1-1-1-1	1-1-1-1	1-1-1-1-1	1-1-1-1	1-1-1

suboptimal values might suggest that it is not worth the effort to find the 'best' possible result by many iterations. Note, however, that these slight differences do not necessarily mean that the differences between the matrices are also negligible. Alternative and almost equally good solutions may exist for the same classification problem; moreover, in some cases the true optimum may not be unique. This important question deserves further investigations.

Acknowledgements. The authors are grateful to the Institute for Environmental and Marine Sciences and Technologies, Trieste, Italy, for research facilities. Comments by N. C. Kenkel are gratefully acknowledged. We are also grateful to M. B. Dale, O. Wildi and J. B. Wilson for their constructive criticism of the manuscript.

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Received 6 December 1990;

Revision received 25 March 1991;

Accepted 25 April 1991;

Final revision received 30 June 1991.

App. 1. Program REBLOCK

Program REBLOCK, written in FORTRAN, for iterative block clustering may be run in batch or interactive mode. Block sharpness is optimized using either of the three criteria described in the paper. The analysis starts from randomly generated or user-specified partitions, and there is an option for constrained block clustering. The program may also be run for simple partitioning of rows (or columns) by defining one cluster for columns (or rows) and selecting the sum of squares or entropy criterion as a measure of within-cluster homogeneity. Percentage contributions of variables, objects and blocks to the block structure are computed. In addition, analysis of concentration (AOCL, Feoli & Orlóci 1979) may be performed on the final blocks to provide ordinations for row and column groups and to facilitate interpretation of results. Since the program may be instructed to skip the iterations, any input matrix may be analyzed by AOCL.

For computers with 640 Kbytes of memory, the program processes up to 300 variables and up to 300 objects with a maximum of 20 groups for each. Labels may be used to identify variables in the output. Data input uses variable FORTRAN format or the more convenient free format.

The IBM PC-DOS and Macintosh versions of the program may be obtained free of charge upon sending a 5.25" or 3.5" double density diskette to the authors. The program is also available through the SYN-TAX IV package (Podani in press).

