## A METHOD FOR CLUSTERING OF BINARY (FLORISTICAL) DATA IN VEGETATION RESEARCH

By

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A new formula, weighted dissimilarity index (WDI) is proposed for measuring local and floral dissimilarity taking the importance of attributes into consideration. Application of this index to the "centroid sorting" method is illustrated by a hypothetical example. Results of the study of a rocky grassland community are also included and briefly discussed.

#### I. Introduction

I.1. It is well-known that the use of binary (presence-absence) data has many advantages in vegetation analysis but in most papers published until now the problem of weighting has generally been neglected. In the comparison of two quadrats all species were taken into consideration of equal weight. It is obvious, however, that some "mass variables" (e.g. frequency) should have some rôles in constructing a sorting algorithm. In this case the more frequent species are more important than the less frequent or rare ones, so the common species should have greater effect on the similarity (or dissimilarity) between two given quadrats. In the same way, in consequence of the duality of attributes, the quadrats of highest species number give the greatest information about some plausible measure of dependence, so they should be considered with a higher weight than the quadrats of small number of species.

Starting from the assumptions mentioned above, I propose a weighted dissimilarity index (denoted by WDI in this paper). The application of WDI seems to have many advantages for cluster analysis, and so it is worth demonstrating by both hypothetical and concrete examples (V—VI).

I.2. The indices and metrics used in vegetation research will not be detailed here. Advantages and disadvantages of these functions have been discussed by numerous papers and monographs (Dagnélie 1960; Sokal and Sneath 1963; Greig-Smith 1964; Kershaw 1964; Cheetham and Hazel 1969; Gounot 1969; Goodall 1973a; Orlógi 1972, 1975 etc.). In this paper only the most important formulae will be mentioned (III.). The methodical problems

of cluster analyses will not be discussed, either, since numerous books and articles are available concerning this topic (Lance and Williams 1966; Williams, Lambert and Lance 1966; Rohlf 1970; Jardine and Sibson 1971; Orlóci 1975; Hartigan 1975).

I.3. The method described in IV.—V. can also be applied to quantitative data (coverage, frequency etc.), but in this study only the binary cases are dealt with. Comparison of quantitative data raises a number of questions the discussion of which is beyond the scope of this paper.

#### II. Definitions

II.1. Before the discussion it is important to give a summary of the denotations and their definitions used in this paper. First let us consider the following matrix:

In matrix  $U^{(0)}$  the columns are the floral vectors of the sampling units, the rows are the local vectors of the species. It is worth mentioning that  $U^{(0)}$  may be called floristic composition which is a set of relations between m sampling units and n species found in the examined area T (Juhász-Nagy 1976).

II.2.  $f_{ij}^{(0)}$  expresses the species-locus relation:

$$f_{ij}^{(0)} \left\{ egin{array}{ll} 1, \ \ ext{if} \ \ s_i \ \ ext{is present in} \ t_j \ 0, \ \ ext{if} \ \ s_i \ \ ext{is not present in} \ \ t_j, \end{array} 
ight.$$

so  $f_{ij}^{(0)}$  is an indicator function. The  $^{(0)}$  upper index indicates that  $U^{(0)}$  is the matrix of the basic data. In the course of the clustering algorithm either the number of the columns or that of the rows of this matrix will be reduced step by step. Then  $f_{ij}^{(x)}$  will no longer be an indicator function and its value will range from 0 to 1 (see V. for more details).

II.3. Let  $v_i$  denote the number of those sampling units in which  $s_i$  is present. Thus v is the valence distribution of all the species:

$$v = [v_1, \dots v_i, \dots v_n], \quad ext{where } v_i = \sum\limits_j f_{ij}^{(0)}.$$
 (II.3.1.)

II.4.  $z_j$  is the number of species present in  $t_j$ . Accordingly z is the valence distribution of the sampling units:

$$z = [z_1, \dots z_j, \dots z_m], \text{ where } z_j = \sum_i f_{ij}^{(0)}$$
 (II.4.1.)

II.5. N denotes the total valence:

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$$N = \sum_{i} v_i = \sum_{j} z_j \tag{II.5.1.}$$

II.6. Let  $\hat{p}_i$  denote the estimated probability of the presence of  $s_i$  in T at the given quadrat size. Thus  $\hat{p}$  is the probability distribution of the species:

$$\hat{p} = [\hat{p}_1, \dots \hat{p}_i, \dots \hat{p}_n,], \quad \text{where } \hat{p}_i = \frac{v_i}{m}$$
 (II.6.1.)

II.7.  $q_j$  denotes the proportion of the species present in  $t_j$ , to the species present in T. Thus q is the floral frequency distribution of the sampling units:

$$q = [q_1, \dots q_j, \dots q_m,], \quad \text{where } q_j = \frac{z_j}{n}$$
 (II.7.1.)

II.8. The following relations are valid for  $p_i$  and  $q_i$ , respectively:

$$0 \le \hat{p}_i \le 1 \tag{II.8.1.}$$

$$0 \le q_j \le 1 \tag{II.8.2.}$$

$$\frac{\mathbf{v}_i}{N} = \frac{\hat{\mathbf{p}}_i}{\sum_i \hat{\mathbf{p}}_i} \tag{II.8.3.}$$

$$\frac{z_j}{N} = \frac{q_j}{\sum_i q_j} \tag{II.8.4.}$$

- II.9. Resemblance functions for binary data use the parameters of the 2\*2 contingency table. In case of a comparison of two quadrats the meanings of the parameters are the following:
- (a) represents the number of common species in the two quadrats in question  $(t_j \text{ and } t_k)$ ;
- (b) is the number of species which occur in quadrat  $t_j$ , but not in  $t_k$ ;
- (c) indicates the number of species which occur in quadrat  $t_k$ , but not in quadrat  $t_j$ ;
- (d) is the number of species absent from quadrats  $t_j$  and  $t_k$ , but present in some other ones.

In calculating the correlation between two species these parameters should be used, of course, in the same way for the pairs of rows of  $U^{(0)}$ .

II.10. Let  $X_{ij}$  denote any quantitative feature (coverage, frequency etc.) of  $s_i$  in  $t_i$ .

## III. Some important coefficients

III.1. Numerous indices for presence-absence data use only the data of the quadrats (or the species) concerned (a, b, c parameters). SØRENSEN index and JACCARD index are well-known examples.

III.2 Other formulae take not only the data of the two compared quadrats into consideration, but those of the rest, too (they are involved in the parameter d). In these cases the number of the joint presence and that of the joint absence has equal effect on resemblance. The most simple of them is the matching coefficient of SOKAL and MICHENER (1958):

$$E_{SM} = \frac{a+d}{a+b+c+d}$$
 (III.2.1.)

The product-moment correlation coefficient is also widely used:

$$E_{\Phi} = \frac{ad - bc}{\sqrt{(a+b)(a+c)(b+d)(c+d)}}$$
(III.2.2.)

Owing to its disadvantages (EADES 1965, BARONI-URBANI and BUSER 1976) this coefficient is proposed for large samples. The formula of BARONI-URBANI and BUSER (1976) can be used for relatively small samples as well without much bias:

$$E_{BB} = \frac{\sqrt{ad + a - b - c}}{\sqrt{ad + a + b + c}}$$
(III.2.3.)

III.3. The absolute value function that has been used for quantitative data only seems to be applicable to the comparison of sampling units:

$$E_{absjk} = \sum_{i} |x_{ij} - x_{ik}|. \qquad (III.3.1.)$$

WHITTAKER (1952) used this formula in its relative form:

$$E_{W_{jk}} = \sum_{i} \left| \frac{\mathbf{x}_{ij}}{\sum_{i} \mathbf{x}_{ij}} - \frac{\mathbf{x}_{ik}}{\sum_{i} \mathbf{x}_{ik}} \right|$$
(III.3.2.)

The similarity between two quadrats is maximal  $(E_W=0)$  if they both contain the same species and the proportion of the species in them is also equal.

The so-called "Canberra-metric" is also based on the absolute differences (LANCE and WILLIAMS 1966):

$$E_{LW_{jk}} = rac{\sum_{i} rac{|x_{ij} - x_{ik}|}{x_{ij} + x_{ik}}}{n}$$
 (III.3.3.)

The value of  $E_{LW}$  ranges from 0 to 1, 0 indicates the maximal similarity. III.4. The coefficients mentioned in III.1.—3. make no distinction between the attributes. Williams—Dale—Macnaughton-Smith (1964) pointed out this insufficiency: "some attributes are more important than others in determining similarity". They proposed a weighted coefficient for quantitative data:

$$E_{WDM_{jk}} = \sum_{i} \left[ (x_{ij} - x_{ik})^2 \cdot \sum_{h \neq i} \chi_{hi}^2 \right],$$
 (III.4.1.)

where  $\sum \chi^2$  is the sum of the  $\chi^2$  values calculated between  $s_i$  and the all other species. When  $\chi^2$  test is applied, however, the same difficulties will arise which appeared at the use of formula III.2.2.

### IV. A new dissimilarity index and its properties

IV.1. In order to eliminate the insufficiences of coefficients having been used for binary cases until now, I propose a new formula, weighted dissimilarity index (WDI), for comparing sampling units defined as

$$d_{jk} = \frac{\sum_{i} (p_i \cdot |f_{ij}^{(0)} - f_{ik}^{(0)}|)}{\sum_{i} p_i}$$
 (IV.1.1.)

The value of  $d_{jk}$  depends on the distinctive species of quadrats  $t_j$  and  $t_k$ . The absolute values of the differences are weighted with the estimated probability of the presence of the species concerned, and then these products are added up. The upper limit of this sum i.e. the numerator of formula IV.1.1. is the quantity  $\sum_i p_i$ , which is the maximum possible dissimilarity between any two quadrats included in  $U^{(0)}$ . It is obvious that  $\sum_i p_i$  is obtained as dissimilarity value if the two compared quadrats neither have joint absence nor joint presence at all. It is reasonable to standardize with the quantity  $\sum_i p_i$ , so the value of  $d_{jk}$  will range from 0 to 1. Since

$$\sum_{i} p_i = \sum_{i} \frac{v_i}{m} = \frac{N}{m}, \qquad (IV.1.2.)$$

the nominator of formula IV.1.1. indicates the average participation of a quadrat in the total valence, i.e. the average number of species in the quadrats.

It is to be noted that  $d_{jk}$  is not directly effected by the number of joint presence and joint absence, so the index IV.1.1. differs essentially from the formulae discussed in III.

IV.2. For measuring the dissimilarity between any pair of species I propose the formula defined as

$$d_{hi} = rac{\sum\limits_{j} (q_{j} \cdot |f_{hj}^{(0)} - f_{ij}^{(0)}|)}{\sum\limits_{j} q_{j}}$$
 (IV.2.1.)

that is analogous to index IV.1.1. The dissimilarity between  $s_h$  and  $s_i$  depends on the number of species of those quadrats in which only one of the two species is present. Hereby the quadrats of a smaller number of species will have less effect on  $d_{hi}$  than the quadrats of a high number of species. It is worth to explain it with a simple example: a quadrat of 10 species contains less information about the interspecific correlations than a quadrat of 35 species, so we must consider the second quadrat with a greater weight.

The quantity of  $\sum_{i} q_{i}$  signifies the average participation of a species in the total valence, i.e. the average number of presence of the species:

$$\sum_{i} q_{j} = \sum_{i} \frac{z_{j}}{n} = \frac{N}{n}.$$
 (IV.2.2.)

- IV.3. The value of d ranges from 0 to 1. Zero indicates minimum disimilarity when all attributes of the two compared sampling units (or species) are the same. "1" indicates maximum dissimilarity which is obtained when the two quadrats (or species) differ from each other in all attributes.
- IV.4. Sometimes it may be more convenient to use the term similarity. Then the values of the similarities between quadrats or species are the following:

$$e_{ik} = 1 - d_{ik} (IV.4.1.)$$

$$e_{hi} = 1 - d_{hi} \tag{IV.4.2.}$$

# V. Application of the weighted dissimilarity index to cluster analysis

- V.1. The matrix of the dissimilarity values denoted by  $D^{(0)}$  may be the starting point of numerous clustering algorithms. A method that regards the clusters as new individuals seems to be most applicable to this case. This is the "centroid sorting" method; its steps were briefly described by WILLIAMS, LAMBERT and LANCE (1966):
- a. The most similar pair of quadrats are added together, attribute by attribute, to form a new synthetic quadrat.
- b. The records of the pair concerned are deleted, together with all coefficients involving either of them.
- c. Coefficients are calculated between the new quadrat and all other remaining quadrats. The process returns to operation a. If all quadrats are fused into a single group, then the analysis is terminated.

V.2. In the course of calculations the columns of the data matrix are reduced by one step by step. Elements of the synthetic quadrat obtained by the fusion of the most similar pair of quadrats are calculated according to the following formula:

 $f_{i(j,k)}^{(x+1)} = \frac{N_j \cdot f_{ij}^{(x)} + N_k \cdot f_{ik}^{(x)}}{N_j + N_k}$ (V.2.1.)

where  $f_{ij}^{(x)}$  and  $f_{ik}^{(x)}$  are the frequencies of species  $s_i$  in the fused quadrats  $t_j$  and  $t_k$  after x fusions. Its value ranges from 0 to 1.  $N_j$  and  $N_k$  are the numbers of quadrats fused in the columns j and k, respectively. If  $N_j = N_k = 1$  the formula V.2.1. reduces to simpler form, for instance in the case of the first fusion:

$$f_{(j)k}^{(1)} = \frac{f_{ij}^{(0)} + f_{ik}^{(0)}}{2}$$
 (V.2.2)

After m-1 fusions the analysis is terminated and  $(m-1)^2$  coefficients are to be calculated. This is the reason why the centroid sorting method, especially in case of large samples, is a computer oriented one.

- V.3. The same algorithm can be used for comparing pairs of species, but in this case the rows of the data matrix are to be fused. The number of fusions will be n-1, the number of coefficients  $(n-1)^2$ .
- V.4. Let this algorithm be demonstrated by a hypothetical example. Let  $U^{(0)}$  consist of 5 columns and 8 rows (5 quadrats and 8 species):

$$U^{(0)} = egin{bmatrix} 1 & 2 & 3 & 4 & 5 \ \hline 1 & 1 & 1 & 0 & 1 & 1 \ 2 & 0 & 0 & 1 & 1 & 1 \ 3 & 1 & 1 & 0 & 0 & 0 \ 4 & 1 & 0 & 0 & 1 & 0 \ 5 & 1 & 1 & 0 & 0 & 0 \ 6 & 0 & 0 & 1 & 1 & 1 \ 7 & 0 & 1 & 1 & 1 & 1 \ 8 & 1 & 1 & 1 & 0 & 0 \ \end{bmatrix}$$

First the distribution  $\hat{p}$  is to be calculated with the help of formula II.6.1., and, then we come to the following result:

$$\hat{p} = 0.8, 0.6, 0.4, 0.4, 0.4, 0.6, 0.8, 0.6$$

Now we calculate matrix  $D^{(0)}$  of the dissimilarity values according to formula IV.1.1. The semimatrix is given by

After the inspection of this matrix we find that the lowest dissimilarity is  $d_{4,5} = 0.086$ , thus we fuse quadrats 4 and 5 to obtain a new synthetic quadrat. We calculate the values of  $f_{i,(4,5)}^{(1)}$  by the application of formula V.2.2. After the first fusion the starting binary data of the quadrats in question will represent new frequency values. The new data matrix is given by

$$U^{(1)} = egin{array}{|c|c|c|c|c|} \hline & 1 & 2 & 3 & (4,5) \\ \hline & 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0.5 \\ 5 & 1 & 1 & 0 & 0 \\ 6 & 0 & 0 & 1 & 1 \\ 7 & 0 & 1 & 1 & 1 \\ 8 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

We compare the synthetic quadrat (4, 5) with all other quadrats and obtain matrix  $D^{(1)}$ :

		1	2	3	(4, 5)
$\mathbf{D}^{(1)} =$	1 2 3 (4, 5)		0.260	0.869 0.608 —	0.782 0.608 0.347

After the first fusion we find that the quadrat 1 and 2 are the most similar pair, since  $d_{1,2} = 0.260$ . We calculate the new column of the data matrix and obtain matrix  $U^{(2)}$ :

Then we calculate matrix  $D^{(2)}$ :

		(1, 2)	3	(4, 5)
$D^{(2)} =$	(1, 2)		0.739	0.652
	J J			0.347
	(4, 5)			

where the lowest dissimilarity value is  $d_{3,(4,5)} = 0.347$ , so we fuse quadrat 3 and (4, 5). After this fusion the reduced data matrix is

		(1, 2)	(3, 4, 5)
$\Pi_{(3)} =$	1 2 3 4 5 6 7 8	1 0 1 0.5 1 0 0.5	0.66 1 0 0.33 0 1 1 0.33

We compare the two columns of  $U^{(3)}$  and obtain matrix  $D^{(3)}$ :

$$\mathbf{D^{(3)}} = \begin{bmatrix} & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & \\ & & \\ & & \\ & & \\ & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\$$

After the last fusion we get vector  $\hat{p}$ :

$$U^{(4)} = \hat{p} = [0.8, 0.6, 0.4, 0.4, 0.4, 0.6, 0.8, 0.6]$$

It is worth to summarize the steps indicating the fusion dissimilarities:

fusion 1	4. 5	d = 0.086
fusion 2	1, 2	d = 0.260
fusion 3	3, (4, 5)	d=0.347
fusion 4	(1, 2), (3, 4, 5)	d = 0.681

The results are illustrated by the dendrogram in Fig. 1.

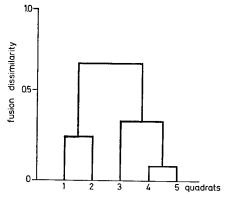


Fig. 1. Dendrogram constructed on the basis of centroid sorting method from data of the hypothetical example discussed in V. 4

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## VI. Application of the WDI to the study of a rocky grassland community

VI.1. The cluster analysis based on the WDI was applied to the study of a rocky grassland community from the Hungarian Central Range, which is considered to be an association named Seslerietum sadlerianae, and has been described according to the methodology of Braun-Blanquet's school (Zólyomi 1936, 1958; Soó 1964). This association is usually found on northern slopes on dolomite rocks (with the exception of Naszály Hill, where the rock is limestone).

33 sampling units of  $4\times 4$  m size from 7 stands of this community were examined. The distribution of the quadrats according to the stands is the following:

Ι.	Sashegy, Buda Hills	quadrats	15.
II.	Hunyadorom, Buda Hills	quadrats	69.
III.	Tündérszikla, Buda Hills	quadrats	10.—14.
IV.	Hármashatárhegy, Buda Hills	quadrats	15.—18.
$\mathbf{V}$ .	Nagykevély, Pilis Hills	quadrats	19.—23.
VI.	Pilistető, Pilis Hills	quadrats	24.—28.
VII.	Naszály Hill	quadrats	2933.

The geographic localization of the stands is shown by Fig. 2.

147 species were found in the quadrats. Their comparison was also performed. Matrix of the presence-absence data is shown in Table 1.

VI.2. Computer program WDCL was written in FORTRAN IV for performing the cluster analysis of sampling units or species. A listing of it is available from the author on request.

VI.3. Dendrogram of the quadrats (Fig. 3) shows that the sampling units are clustered according to the stands. The only exception is quadrat 12.

At the level of d=0.354 the quadrats from the stand of Naszály Hill differ from the rest of quadrats. This fact supports the view, that the stand of Naszály Hill represents a special type of this community. It was originally described as a "subassociation" named Seslerietum sadlerianae saxifragetosum aizoi (for details see Zólyomi 1958; Soó 1964). This result was also obtained by the use of the association-analysis worked out by Williams and Lambert (Podani 1976).

At the next level the cluster of quadrats 12, 15–18, 24–28 differ from the remaining quadrats. This level (d=0.27) is lower than the dissimilarity between the cluster of quadrats 12., 24.—28. and the cluster of quadrats 15–18 (d=0.299). Here we can find some other "monotonicity failures",\* too (it is to be noted that in the cluster of species there are only two monotonicity failures, see Fig. 4). These failures are caused by the centroid

<sup>\*</sup> In the sense of LANCE and WILLIAMS 1966.

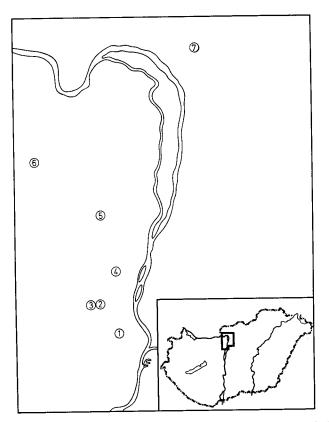


Fig. 2. Geographic localization of the stands of Seslerietum sadlerianae included in the study

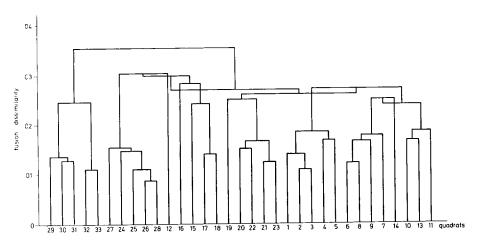


Fig. 3. Dendrogram indicating hierarchy for quadrats in Table 1. See text

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Naszály	1 1		_
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	per	1. Sesleria sadleriana 2. Sanguisorba minor 3. Draba lasiocarpa 4. Alyssum montanum 6. Alyssum montanum 7. Authorna mollis 8. Festuca pallens 9. F. sulcata 10. Sesli leucospermum 11. S. osseum 12. S. hippomarathrum 13. Carex humilis 14. C. liparocarpos 15. Thymus praecox 16. T. glabrescens 17. Globularia aphyllant 18. Polygonatum odorati 19. Dianthus regisstepha 10. Dianthus regisstepha 11. Globularia aphyllant 12. Helianthemum canu 13. Houtum 13. Houtum 14. Centaurea sadleriana 15. C. micranthos 16. C. riumfettii 17. Scabiosa ochroleuca 18. S. canescens 18. S. canescens 18. S. canescens 19. Thesium linophylloi 10. Asperula glauca 11. Ascharicum remosum 12. Stachys recta	nn
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sorting method itself, which does not exclude the possibility that the dissimilarity between two clusters is lower than between the quadrats fused in one of these clusters. This feature may be useful with respect to further considerations (besides, it does not reduce very much the clearness of the dendrogram).

Now I will not go into fruther details concerning the dendrogram of the quadrats. The relationships are shown by Fig. 3.

VI.4. Dendrogram of the species is illustrated in Fig. 4. According to this figure the species can be divided into two main groups (d=0.553). The group consisting of 20 members contains the most frequent and perhaps the "most typical" species of the community. These species are absent from not more than 1-2 stands. These are the following (nomenclature of species follows Soó and Kárpáti 1968):

Carex humilis (13) and Polygonatum odoratum (18) are absent from Naszály Hill.

Scabiosa ochroleuca (27), Allium flavum (49), Helianthemum ovatum (22) and Euphorbia cyparissias (60) are absent from Sashegy.

Cytisus hirsutus (61) is absent from Nagykevély.

Asperula glauca (30) is the only species of this group which is absent from two stands (Hunyadorom and Nagykevély).

In the left part of the cluster of the other species group (from species 64 to species 117) we can find numerous frequent species but they are absent from 2-4 stands at least, and most of them are typical only in 2-3 stands. It is worth mentioning the case of Festuca pallens (8) which occurs in all quadrats of 5 stands but it is absent from 2 stands so it did not get into the group of typical species. There is only one species in this group, Sempervivum hirtum (52), which is present in all stands.

The rare species are clustered in the rightmost part of the dendrogram. A number of them are either typical in one stand (from species 28 to species 123) or present in 1-2 quadrats (from species 77 to species 120). It is striking that the values of dissimilarities are lower between the rare species than between the frequent ones. These low values, however, may be caused by some accidental effects which influence very much the dissimilarities between rare species. Thus, we can not draw general conclusions from these results. The situation is rather different in the case of frequent species although the dissimilarity values are relatively higher. But it is likely that the group of 20 species can also be found in stands that were not included in the analysis.

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#### REFERENCES

- BARONI-URBANI, C.—BUSER, M. W. (1976): Similarity of binary data. Syst. Zool. 25, 251—259. CHEETHAM, A. H.—HAZEL, J. E. (1969): Binary (presence-absence) similarity coefficients. J. Paleont. 43, 1130-1136.
- DAGNÉLIE, P. (1960): Contribution à l'étude des communautés végétales par l'analyse factorielle. Bull. Serv. Carte Phytogéogr. (B), 5, 7-71, 93-195.
- EADES, D. C. (1965): The inappropriateness of the correlation coefficient as a measure of taxonomic resemblance. Syst. Zool. 14, 98-100.
- GOODALL, D. W. (1973a): Sample similarity and species correlation. In: WHITTAKER (1973), 107 - 156.
- GOODALL, D. W. (1973b): Numerical classification. In: WHITTAKER (1973), 577-615.
- GOUNOT, M. (1969): Méthodes d'étude quantitative de la végétation. MASSON, Paris.
- GREIG-SMITH, P. (1964): Quantitative Plant Ecology. 2nd ed. BUTTERWORTHS, London.
- HARTIGAN, J. A. (1975): Clustering Algorithms. WILEY, New York.

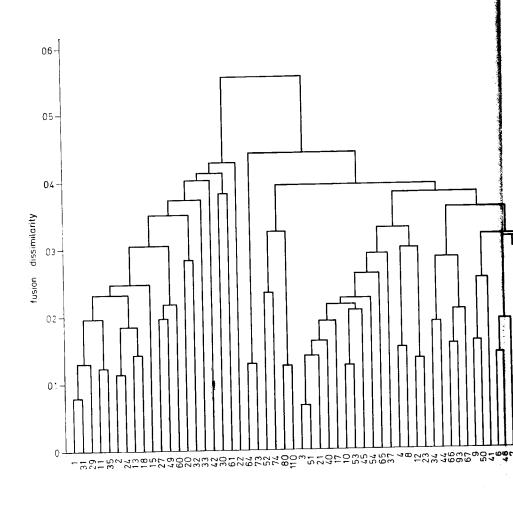
  JARDINE, N.—Sibson, R. (1971): Mathematical Taxonomy. WILEY, New York.
- JARDINE, N.—SIBSON, R. (1971): Mathematical Taxonomy. WILEY, New YORK.

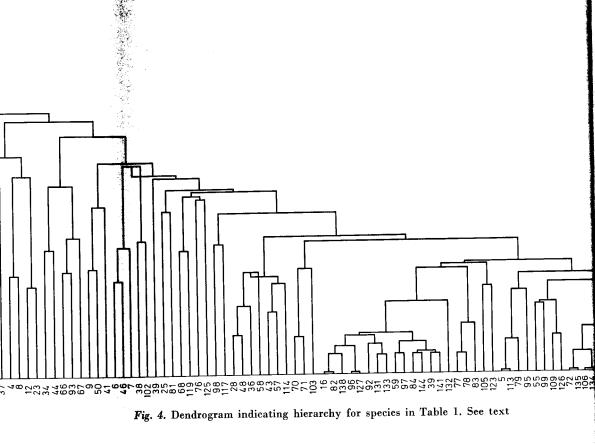
  JUHÁSZ-NAGY, P. (1976): Spatial dependence of plant populations. Part I. Equivalence analysis
  (an outline for a new model). Acta Bot. Acad. Sci. Hung. 22, 61—78.

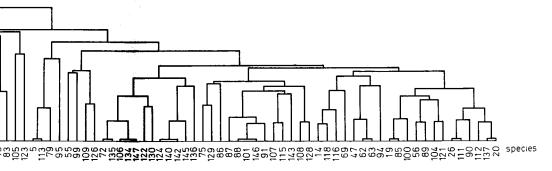
  KERSHAW, K. A. (1964): Quantitative and Dynamic Ecology. Edward Arnold, London.

  LANCE, G. N.—WILLIAMS, W. T. (1966): Computer programs for hierarchical polythetic classification ("similarity analyses"). Comput. J. 9, 60—64.

  MACNAUGHTON-SMITH, P.—WILLIAMS, W. T.—DALE, M. B.—MOCKETT, L. G. (1964): "Dissimilarity analysis: a new technique of hierarchical subdivision". Nature 202, 1034.
- Orlóci, L. (1972): On objective functions of phytosociological resemblance. Am. Midland Nat. 88, 28-55.
- ORLÓCI, L. (1975): Multivariate Analysis in Vegetation Research. JUNK, The Hague.
- Podani, J. (1976): A budai nyúlfarkfüves sziklagyepek cönológiai vizsgálata (Cenological study of Seslerietum sadlerianae). M. A. thesis (manuscript).
- ROHLF, F. J. (1970): Adaptive hierarchical clustering schemes. Syst. Zool. 18, 58-82.
- SOKAL, R. R.—MICHENER, C. D. (1958): A statistical method for evaluating systematic relationship. Univ. Kansas. Sci. Bull. 38, 1409—1438 (cit. in Sokal—Sneath 1963).
- Sokal, R. R. Sneath, P. H. A. (1963): Principles of numerical taxonomy. Freeman, San Francisco.
- Soó, R. (1964): A magyar flóra és vegetáció rendszertani-növényföldrajzi kézikönyve I. (Synopsis systematico-geobotanica florae vegetationisque Hungariae I). Akadémiai Kiadó, Budapest.
- Soó, R.-Kárpáti, Z. (1968): Növényhatározó. Tankönyvkiadó, Budapest.
- WEBSTER, R.-Burrough, P. A. (1972): Computer-based soil mapping of small areas from sample data I. Multivariate classification and ordination. J. Soil Sci. 23, 210-221.
- WHITTAKER, R. H. (1952): A study of summer foliage insect communities in the Great Smoky Mountains. Ecol. Monogr. 22, 1-44.
- WHITTAKER, R. H. ed. (1973): Ordination and classification of vegetation (Handbook of Vegetation Science, part V.). Junk, The Hague.
- WILLIAMS, W. T.-DALE, M. B.-MACNAUGHTON-SMITH, P. (1964): An Objective Method of Weighting in Similarity Analysis. Nature 201, 426.
  WILLIAMS, W. T.-LAMBERT, J. M.-LANCE, G. N. (1966): Multivariate methods in plant
- ecology V. Similarity analyses and information-analysis. J. Ecol. 54, 427-445.
- Zólyomi, B. (1936): A pannóniai flóratartomány és az északnyugatnak határos területek sziklanövényzetének áttekintése (Übersicht der Felsenvegetation in der pannonischen Florenprovinz und dem nordwestlich angrenzenden Gebiete). Ann. Hist.-Nat. Mus. Hung. 30, 136-174.
- Zólyomi, В. (1958): Budapest és környékének természetes növénytakarója (The natural vegetation of Budapest and its surroundings). In: Budapest Természeti Képe, Budapest. 508 - 642.







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